EE 508
Lecture 21

Sensitivity Functions

- Comparison of Circuits
- Predistortion and Calibration

Review Correction from last time

Theorem: If all op amps in a filter are ideal, then ω_o, Q, BW, all band edges, and all poles and zeros are homogeneous of order 0 in the impedances.

Theorem: If all op amps in a filter are ideal and if T(s) is a dimensionless transfer function, $T(s)$, $T(j\omega)$, $|T(j\omega)|$, $\angle T(j\omega)$, are homogeneous of order 0 in the impedances

Review from last time

Bilinear Property of Electrical Networks

Theorem: Let x be any component or Op Amp time constant (1^{st} order Op Amp model) of any linear active network employing a finite number of amplifiers and lumped passive components. Any transfer function of the network can be expressed in the form

$$T(s)=\frac{N_0(s)+xN_1(s)}{D_0(s)+xD_1(s)}$$

where N_0, N_1, D_0, and D_1 are polynomials in s that are not dependent upon x

A function that can be expressed as given above is said to be a bilinear function in the variable x and this is termed a bilateral property of electrical networks.

The bilinear relationship is useful for

1. Checking for possible errors in an analysis
2. Pole sensitivity analysis

Review from last time

Root Sensitivities

Consider expressing T(s) as a bilinear fraction in x

$$T(s)=\frac{N_0(s)+xN_1(s)}{D_0(s)+xD_1(s)}=\frac{N(s)}{D(s)}$$

Theorem: If z_i is any simple zero and/or p_i is any simple pole of T(s), then

$$S_x^{z_i}=\left(\frac{x}{z_i}\right)\left(\frac{-N_1(z_i)}{\frac{dN(z_i)}{dz_i}}\right) \quad \text{and} \quad S_x^{p_i}=\left(\frac{x}{p_i}\right)\left(\frac{-D_1(p_i)}{\frac{dD(p_i)}{dp_i}}\right)$$

Note: Do not need to find expressions for the poles or the zeros to find the pole and zero sensitivities !

Note: Do need the poles or zeros but they will generally be known by design

Note: Will make minor modifications for extreme values for x (i.e. τ for op amps)

Review from last time

Root Sensitivities

Summary: Pole (or zero) locations due to component variations can be approximated with simple analytical calculations without obtaining parametric expressions for the poles (or zeros).

$$p_i \simeq p_i\Big|_{\substack{\text{Ideal}\\\text{Components}}} + \Delta p_i$$

where

$$\Delta p_i \simeq \Delta x \bullet \mathcal{S}_x^{p_i}$$

$$\mathcal{S}_x^{p_i} = -\frac{D_1(p_i)}{\left.\left(\dfrac{\partial D(p_i)}{\partial p_i}\right)\right|_{p_{iN}}}$$

and

$$D(s) = D_0(s) + x \bullet D_1(s)$$

Alternately,

$$\Delta p_i \simeq \left(|p_i|\frac{\Delta x}{x}\right)\tilde{S}_x^{p_i}$$

Review from last time

Transfer Function Sensitivities

$$S_x^{T(s)}\Big|_{s=j\omega} = S_x^{T(j\omega)}$$

$$S_x^{T(j\omega)} = S_x^{|T(j\omega)|} + j\theta S_x^{\theta}$$ where $\theta = \angle T(j\omega)$

$$S_x^{|T(j\omega)|} = \mathrm{Re}\left(S_x^{T(j\omega)}\right)$$

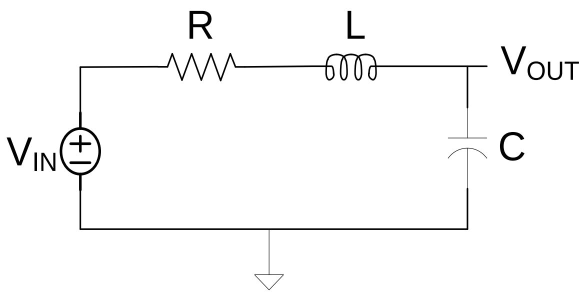
$$S_x^{\theta} = \frac{1}{\theta}\mathrm{Im}\left(S_x^{T(j\omega)}\right)$$

Review from last time

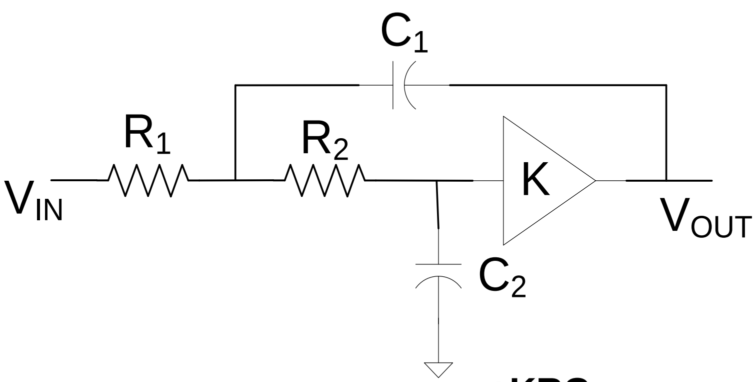
Sensitivity Comparisons

Consider 5 second-order lowpass filters

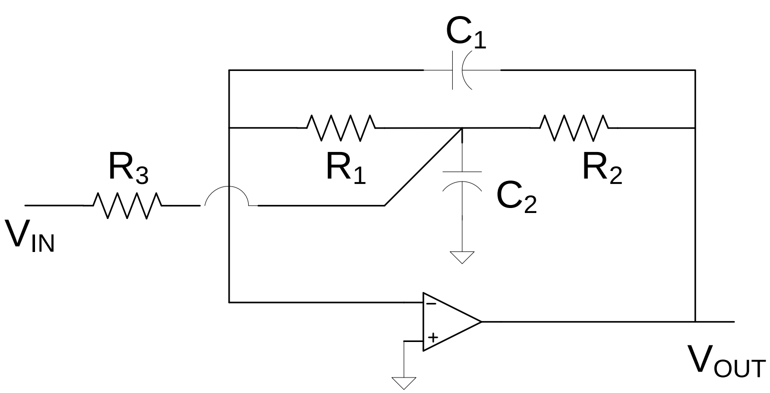
(all can realize same T(s) within a gain factor)



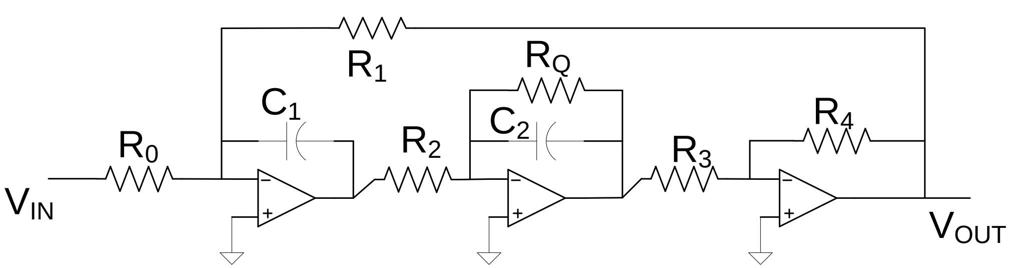
Passive RLC



+KRC



Bridged-T Feedback



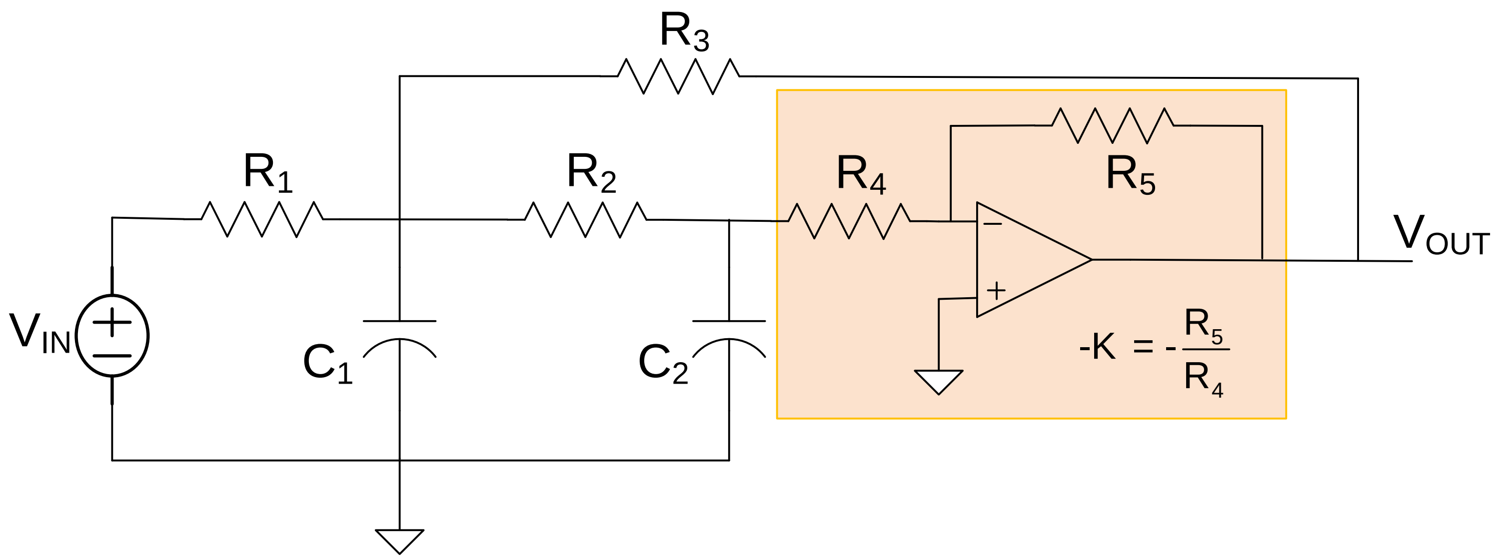
Two-Integrator Loop

Review from last time

Sensitivity Comparisons

Consider 5 second-order lowpass filters

(all can realize same T(s) within a gain factor)



-KRC Lowpass

How do these five circuits compare?

a) From a passive sensitivity viewpoint?

- If Q is small
- If Q is large

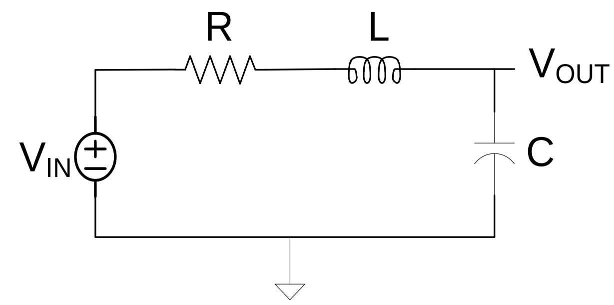
b) From an active sensitivity viewpoint?

- If Q is small
- If Q is large
- If $\tau\omega_0$ is large

Comparison: Calculate all ω_0 and Q sensitivities

Consider passive sensitivities first

a) – Passive RLC



$$S_R^{\omega_0} = 0$$

$$S_L^{\omega_0} = -\frac{1}{2}$$

$$S_C^{\omega_0} = -\frac{1}{2}$$

$$S_R^{Q} = -1$$

$$S_C^{Q} = -\frac{1}{2}$$

$$S_L^{Q} = \frac{1}{2}$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

Case b1 : +KRC Equal R, Equal C

$$\omega_0 = \sqrt{\frac{1}{R_1R_2C_1C_2}}$$

$$Q = \frac{1}{\left(\sqrt{\frac{R_1C_1}{R_2C_2}} + \sqrt{\frac{R_2C_2}{R_1C_1}} + \sqrt{\frac{R_1C_2}{R_2C_1}} - K\sqrt{\frac{R_1C_1}{R_2C_2}}\right)}$$

$$S_{R_1}^{\omega_0} = S_{R_2}^{\omega_0} = S_{C_1}^{\omega_0} = S_{C_2}^{\omega_0} = -\frac{1}{2}$$

$$S_K^{\omega_0} = 0$$

$$S_{R_1}^{Q} = Q - \frac{1}{2}$$

$$S_{R_2}^{Q} = -Q + \frac{1}{2}$$

$$S_{C_1}^{Q} = 2Q - \frac{1}{2}$$

$$S_{C_2}^{Q} = -2Q + \frac{1}{2}$$

$$S_K^{Q} = 3Q - 1$$

$$Q = \frac{1}{3 - K}$$

$$\omega_0 = \frac{1}{RC}$$

If $Q_N = 10$, what happens to Q if

R_1 increases by 1% ?

10% ?

$\frac{\Delta R_1}{R_1} = .01$

$$\frac{\Delta Q}{Q} \approx S^{Q}_{R_1} \cdot \frac{\Delta R_1}{R_1} \approx (Q - 1/2)(.01) = .095$$

$\therefore$ Q changes by 9.5%

$\frac{\Delta R_1}{R_1} = 0.1$

$$\frac{\Delta Q}{Q} \approx S^{Q}_{R_1} \cdot \frac{\Delta R_1}{R_1} = (9.5)(.1) = .95$$

$\therefore$ Q changes by 95%

Actual: 10 $\rightarrow$ 11.04 for $\frac{\Delta R}{R} = .01$

10 $\rightarrow$ 105 for $\frac{\Delta R}{R} = 0.1$

Case b2 : +KRC Equal R, K=1

$$\omega_0 = \sqrt{\frac{1}{R_1R_2C_1C_2}}$$

$$Q = \frac{1}{\left(\sqrt{\frac{R_1C_1}{R_2C_2}}+\sqrt{\frac{R_2C_2}{R_1C_1}}+\sqrt{\frac{R_1C_2}{R_2C_1}}-K\sqrt{\frac{R_1C_1}{R_2C_2}}\right)}$$

$$S_{R_1}^{\omega_0} = S_{R_2}^{\omega_0} = S_{C_1}^{\omega_0} = S_{C_2}^{\omega_0} = -\frac{1}{2}$$

$$S_K^{\omega_0} = 0$$

$$S_{R_1}^{Q} = 0$$

$$S_{R_2}^{Q} = 0$$

$$\omega_0 = \frac{1}{RC}$$

$$S_{C_1}^{Q} = \frac{1}{2}$$

$$S_{C_2}^{Q} = -\frac{1}{2}$$

$$Q = \frac{1}{2}\sqrt{\frac{C_1}{C_2}}$$

$$S_K^{Q} = 2Q^2$$

c) Bridged T Feedback

$$\omega_0 = \sqrt{\frac{1}{R_1R_2C_1C_2}}$$

$$Q = \frac{1}{\left(\sqrt{\frac{C_2}{C_1}}\right)\left(\sqrt{\frac{R_1}{R_3}}+\sqrt{\frac{R_2}{R_1}}+\frac{\sqrt{R_1R_2}}{R_3}\right)}$$

For $R_1=R_2=R_3=R$

$$S_{R_1}^{\omega_0} = S_{R_2}^{\omega_0} = S_{C_1}^{\omega_0} = S_{C_2}^{\omega_0} = -\frac{1}{2}$$

$$S_{R_3}^{\omega_0} = 0$$

$$S_{R_1}^{Q} = -\frac{1}{6}$$

$$S_{R_2}^{Q} = -\frac{1}{6}$$

$$S_{R_3}^{Q} = \frac{1}{3}$$

$$S_{C_1}^{Q} = -\frac{1}{2}$$

$$S_{C_2}^{Q} = \frac{1}{2}$$

$$\omega_0 = \frac{3Q}{RC_1}$$

$$Q = \frac{1}{3}\sqrt{\frac{C_1}{C_2}}$$

d) 2 integrator loop

$$\omega_0 = \sqrt{\frac{R_4}{R_3} \bullet \frac{1}{R_0 R_2 C_1 C_2}}$$

$$Q = \frac{R_Q}{\sqrt{R_0 R_2}} \sqrt{\frac{C_2}{C_1}}$$

For: $R_0 = R_1 = R_2 = R$ $\quad C_1 = C_2 = C$ $\quad R_3 = R_4$

$$S^{\omega_0}_{R_1} = S^{\omega_0}_{R_2} = S^{\omega_0}_{R_3} = S^{\omega_0}_{C_1} = S^{\omega_0}_{C_2} = -\frac{1}{2}$$

$$S^{\omega_0}_{R_4} = \frac{1}{2}$$

$$S^{Q}_{R_1} = S^{Q}_{R_2} = S^{Q}_{R_3} = S^{Q}_{C_1} = -\frac{1}{2}$$

$$S^{Q}_{R_4} = S^{Q}_{C_2} = \frac{1}{2}$$

$$\omega_0 = \frac{1}{RC}$$

$$S^{Q}_{R_Q} = 1$$

$$Q = \frac{R_Q}{R}$$

$$S^{Q}_{R_0} = 0$$

d) -KRC passive sensitivities

$$\omega_0 = \sqrt{\frac{1+(R_1/R_3)(1+K)+(R_1/R_4)(1+R_2/R_3+R_2/R_1)}{R_1R_2C_1C_2}}$$

$$Q = \frac{\sqrt{\dfrac{1+(R_1/R_3)(1+K)+(R_1/R_4)(1+R_2/R_3+R_2/R_1)}{R_1R_2C_1C_2}}}{\left(1+\dfrac{R_1}{R_3}\right)\left(\dfrac{1}{R_1C_1}\right)+\left(1+\dfrac{C_2}{C_1}\right)\left(\dfrac{1}{R_2C_2}\right)+\left(\dfrac{1}{R_4C_2}\right)}$$

For $R_1=R_2=R_3=R_4=R$, $C_1=C_2=C$

$$Q = \frac{\sqrt{5+K_0}}{5} \qquad \omega_0 = \frac{\sqrt{5+K}}{R\ C}$$

$$S_{R_1}^{\omega_0} = -\frac{1}{25Q^2} \qquad S_{R_2}^{\omega_0} = -\frac{1}{2}+\frac{1}{25Q^2} \qquad S_{R_3}^{\omega_0} = -\frac{1}{2}+\frac{3}{50Q^2}$$

$$S_{C_1}^{\omega_0} = S_{C_2}^{\omega_0} = -\frac{1}{2} \qquad S_{R_4}^{\omega_0} = -\frac{3}{50Q^2} \qquad S_{K}^{\omega_0} = \frac{1}{2}+\frac{1}{10Q^2}$$

$$S_{R_1}^{Q} = \frac{1}{5}-\frac{1}{25Q^2} \qquad S_{R_2}^{Q} = -\frac{1}{10}+\frac{1}{25Q^2} \qquad S_{R_3}^{Q} = -\frac{3}{10}+\frac{3}{50Q^2}$$

$$S_{R_4}^{Q} = \frac{1}{5}-\frac{3}{50Q^2} \qquad S_{C_2}^{Q} = -\frac{1}{10} \qquad S_{C_1}^{Q} = \frac{1}{10} \qquad S_{K}^{Q} = \frac{1}{2}-\frac{1}{10Q^2}$$

Passive Sensitivity Comparisons

	$\left\| S_x^{\omega_0} \right\|$	$\left\| S_x^{Q} \right\|$
Passive RLC	$\leq \frac{1}{2}$	**1,1/2**
+KRC		
Equal R, Equal C (K=3-1/Q)	**0,1/2**	**Q, 2Q, 3Q**
Equal R, K=1 ($C_1=4Q^2C_2$)	**0,1/2**	**0,1/2, $2Q^2$**
Bridged-T Feedback	**0,1/2**	**1/3,1/2, 1/6**
Two-Integrator Loop	**0,1/2**	**1,1/2, 0**
-KRC	**less than or equal to 1/2**	**less than or equal to 1/2**

Substantial Differences Between (or in) Architectures

How do active sensitivities compare?

$$S_{k}^{\omega_0} = ? \qquad S_{k}^{Q} = ?$$

Recall $S_x^f = \frac{\partial f}{\partial x}\frac{x}{f}$

so $\frac{\Delta f}{f} \simeq \frac{\Delta x}{x} S_x^f$

but if x is ideally 0, not useful

$$\mathcal{S}_x^f = \frac{\partial f}{\partial x}$$

$$\frac{\Delta f}{f} \simeq \mathcal{S}_x^f \frac{\Delta x}{f}$$

Where we are at with sensitivity analysis:

Considered a group of five second-order filters

Passive Sensitivity Analysis

- Closed form expressions were obtained for ω_0 and Q
- Tedious but straightforward calculations provided passive sensitivities directly from the closed form expressions ???

Active Sensitivity Analysis

- Closed form expressions for ω_0 and Q are very difficult or impossible to obtain

If we consider higher-order filters

Passive Sensitivity Analysis

- Closed form expressions for ω_0 and Q are very difficult or impossible to obtain for many useful structures

Active Sensitivity Analysis

- Closed form expressions for ω_0 and Q are very difficult or impossible to obtain

Need some better method for obtaining sensitivities when closed-form expressions are difficult or impractical to obtain or manipulate !!

Relationship between pole sensitivities and ω_0 and Q sensitivities

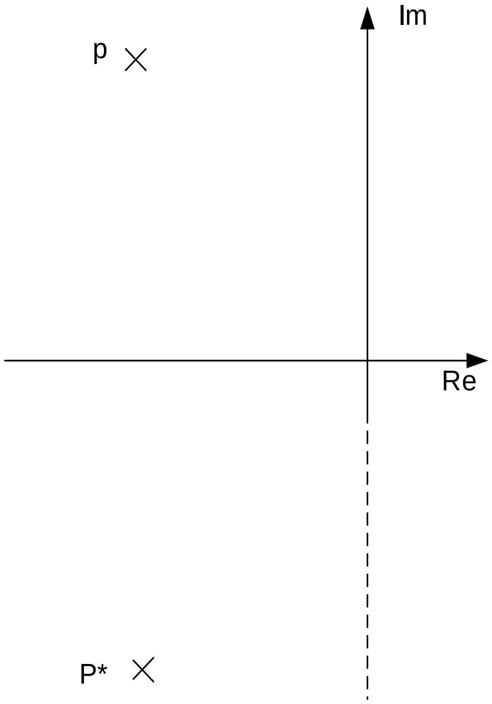
$p = -\alpha + j\beta$

$D_2(s) = (s-p)(s-p^*)$

$D_2(s) = (s+\alpha-j\beta)(s+\alpha+j\beta)$

$D_2(s) = s^2 + s(2\alpha) + (\alpha^2+\beta^2)$

$$D_2(s) = s^2 + s\frac{\omega_0}{Q} + \omega_0^2$$



Relationship between active pole sensitivities and ω_0 and Q sensitivities

Define $D(s)=D_0(s)+t\,D_1(s)$ (from bilinear form of T(s))

Recall: $$s_\tau^p = \frac{-D_1(p)}{\left.\frac{\partial D(s)}{\partial s}\right|_{s=p,\tau=0}}$$

Theorem: $$\Delta p \cong \tau s_\tau^p$$

Theorem: $$\Delta\alpha \cong \tau \operatorname{Re}\left(s_\tau^p\right)$$
$$\Delta\beta \cong \tau \operatorname{Im}\left(s_\tau^p\right)$$

Theorem:

$$\frac{\Delta\omega_0}{\omega_0} \cong \frac{1}{2Q}\frac{\Delta\alpha}{\omega_0} + \sqrt{1-\frac{1}{4Q^2}}\,\frac{\Delta\beta}{\omega_0}$$

$$\frac{\Delta Q}{Q} \cong -2Q\left(1-\frac{1}{4Q^2}\right)\frac{\Delta\alpha}{\omega_0} + \sqrt{1-\frac{1}{4Q^2}}\,\frac{\Delta\beta}{\omega_0}$$

Claim: These theorems, with straightforward modification, also apply to other parameters (R, C, L, K, …) where, $D_0(s)$ and $D_1(s)$ will change since the parameter is different

Active Sensitivities

+KRC

Equal-R, Equal-C

$$\omega_o = \frac{1}{RC}, \qquad Q = \frac{1}{3 - K_o}$$

$$\frac{V_o}{V_i} = \frac{\left(3 - \frac{1}{Q}\right)\omega_o^2}{s^2 + s\frac{\omega_o}{Q} + \omega_o^2 + \frac{\left(3 - \frac{1}{Q}\right)}{\text{GB}} s(s^2 + s3\omega_o + \omega_o^2)} \qquad \left(\omega_o \ll \frac{\omega_o}{2Q}\right)$$

$$-\frac{\Delta\alpha}{\omega_o} \cong \frac{1}{2Q}\left(3 - \frac{1}{Q}\right)^2 \frac{\omega_o}{\text{GB}}, \qquad \frac{\Delta\beta}{\omega_o} \cong -\frac{1}{2}\left(3 - \frac{1}{Q}\right)^2 \frac{\left(1 - \frac{1}{2Q^2}\right)}{\sqrt{1 - \frac{1}{4Q^2}}} \frac{\omega_o}{\text{GB}}$$

$$\frac{\Delta\omega_o}{\omega_o} \cong -\frac{1}{2}\left(3 - \frac{1}{Q}\right)^2 \frac{\omega_o}{\text{GB}}, \qquad \frac{\Delta Q}{Q} \cong \frac{1}{2}\left(3 - \frac{1}{Q}\right)^2 \frac{\omega_o}{\text{GB}}$$

Unity-gain, Equal-R

$$\omega_o = \frac{1}{R\sqrt{C_1 C_2}}, \qquad Q = \frac{1}{2}\sqrt{\frac{C_1}{C_2}}$$

$$\frac{V_o}{V_i} = \frac{\omega_o^2}{s^2 + s\frac{\omega_o}{Q} + \omega_o^2 + \frac{s}{\text{GB}}\left[s^2 + s\omega_o\left(2Q + \frac{1}{Q}\right) + \omega_o^2\right]} \qquad \left(\omega_o \ll \frac{\omega_o}{2Q}\right)$$

$$-\frac{\Delta\alpha}{\omega_o} \cong \frac{\omega_o}{\text{GB}}, \qquad \frac{\Delta\beta}{\omega_o} \cong -Q\frac{\left(1 - \frac{1}{2Q^2}\right)}{\sqrt{1 - \frac{1}{4Q^2}}} \frac{\omega_o}{\text{GB}}$$

$$\frac{\Delta\omega_o}{\omega_o} \cong -Q\frac{\omega_o}{\text{GB}}, \qquad \frac{\Delta Q}{Q} \cong Q\frac{\omega_o}{\text{GB}}$$

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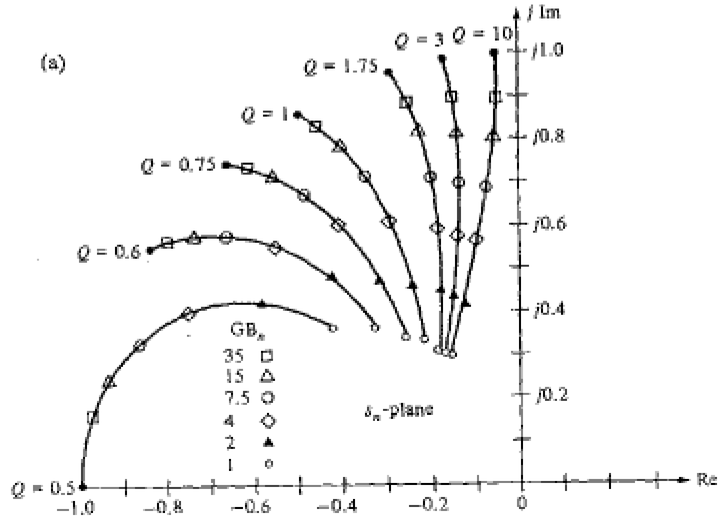
where

$$s_n = \frac{s}{\omega_o}, \qquad \text{GB}_n = \frac{\text{GB}}{\omega_o}.$$

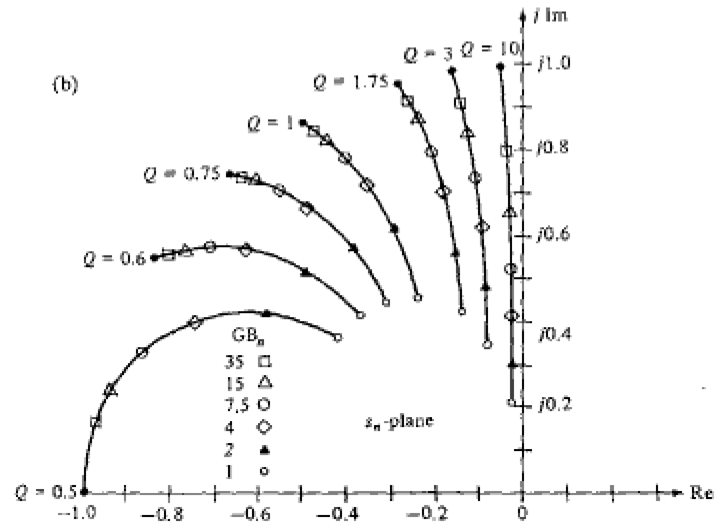
Active Sensitivities

+KRC



◀Fig. 10-5a Plot of upper half-plane root of

$$s_n^3 + s_n^2\left(3 + \frac{QGB_n}{3Q-1}\right) + s_n\left(1 + \frac{GB_n}{3Q-1}\right) + \frac{QGB_n}{3Q-1} = 0 \qquad \text{(Equal-}R\text{, equal-}C\text{)}$$



◀Fig. 10-5b Plot of upper half plane root of

$$s_n^3 + s_n^2\left(2Q + \frac{1}{Q} + GB_n\right) + s_n\left(1 + \frac{GB_n}{Q}\right) + GB_n = 0 \qquad \text{(Unity-gain, equal-}R\text{)}$$

Active Sensitivities

Bridged T Feedback

Table 10-3 Infinite-gain Realization
(see Fig. 10-10b)

Equal-R

$$\omega_o = \frac{1}{R\sqrt{C_1 C_2}}; \qquad Q = \frac{1}{3}\sqrt{\frac{C_1}{C_2}}$$

$$\frac{V_o}{V_i} = -\frac{\omega_o^2}{s^2 + s\frac{\omega_o}{Q} + \omega_o^2 + \frac{s}{\mathrm{GB}}\left[s^2 + s\omega_o\left(3Q + \frac{1}{Q}\right) + 2\omega_o^2\right]} \qquad \left(\omega_o \ll \frac{\omega_a}{2Q}\right)$$

$$-\frac{\Delta\alpha}{\omega_o} \simeq \frac{\omega_o}{\mathrm{GB}}, \qquad \frac{\Delta\beta}{\omega_o} \simeq -\frac{1}{2}\,\frac{3Q - \frac{1}{Q}}{\sqrt{1 - \frac{1}{4Q^2}}}\,\frac{\omega_o}{\mathrm{GB}}$$

$$\frac{\Delta\omega_o}{\omega_o} \simeq -\frac{3Q}{2}\,\frac{\omega_o}{\mathrm{GB}}, \qquad \frac{\Delta Q}{Q} \simeq \frac{Q}{2}\,\frac{\omega_o}{\mathrm{GB}}$$

Active Sensitivities

Bridged T Feedback

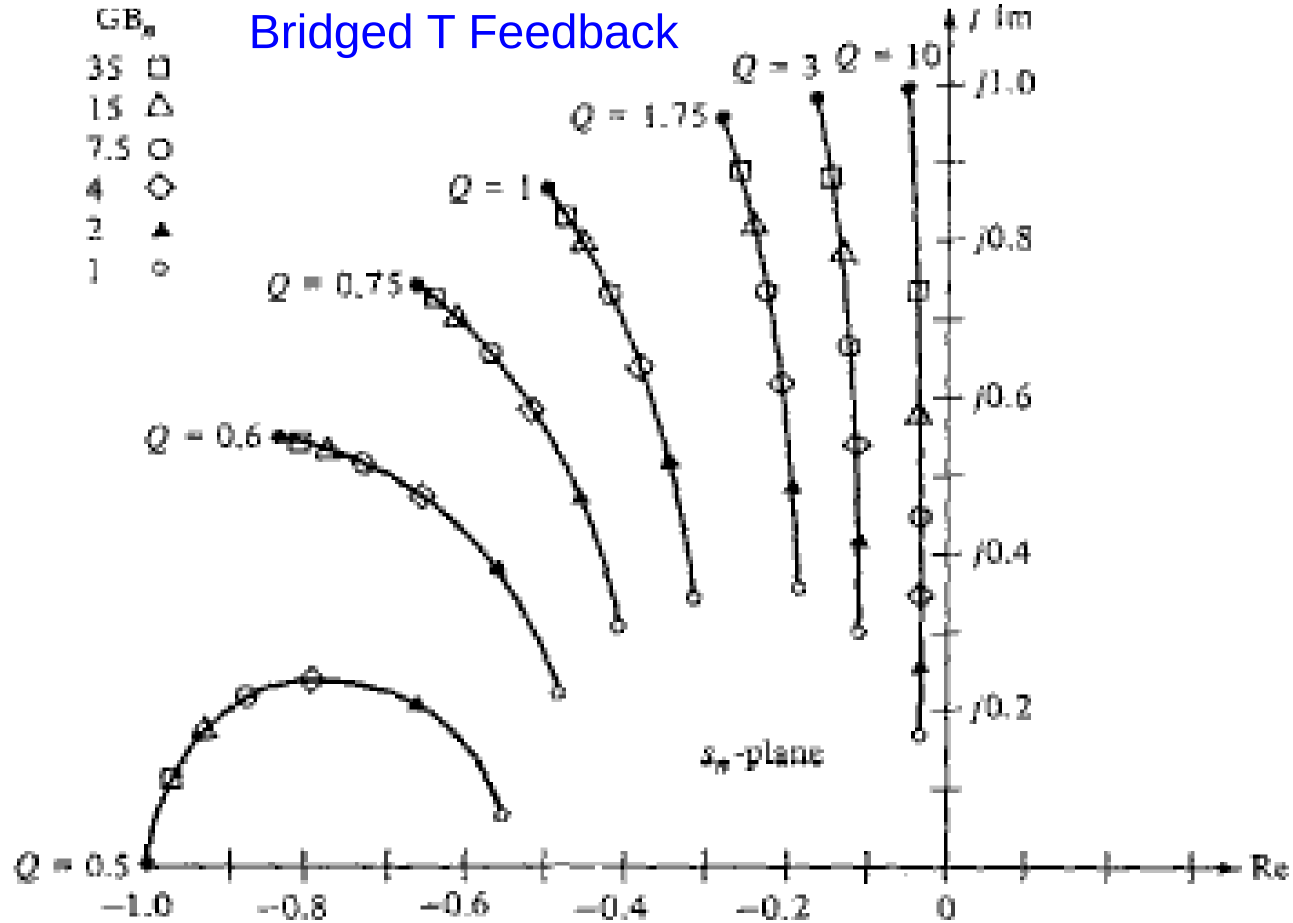


Fig. 10-12 Plot of upper half-plane root of

$$s_n^3 + s_n^2\left(3Q + \frac{1}{Q} + \mathrm{GB}_n\right) + s_n\left(2 + \frac{\mathrm{GB}_n}{Q}\right) + \mathrm{GB}_n = 0$$

Active Sensitivities

Two integrator loop architecture

Equal-R (except R_Q) and Equal-C

$$\omega_o = \frac{1}{RC}, \qquad Q = \frac{R_Q}{R}$$

$$\frac{V_B}{V_i} \cong \frac{\omega_o^2\left(\frac{2}{\mathrm{GB}}s + 1\right)}{s^2 + s\frac{\omega_o}{Q} + \omega_o^2 + \frac{1}{\mathrm{GB}}\left\{4s\left[s^2 + s\omega_o\left(\frac{1}{2} + \frac{1}{Q}\right) + \frac{\omega_o^2}{4Q}\right]\right\}} \qquad \left(\omega_o \ll \frac{\omega_a}{2Q}\right)$$

$$-\frac{\Delta\alpha}{\omega_o} \simeq 2\left(1 + \frac{1}{4Q}\right)\frac{\omega_o}{\mathrm{GB}}, \qquad \frac{\Delta\beta}{\omega_o} \cong -\frac{\left(1 - \frac{1}{Q} - \frac{1}{4Q^2}\right)}{\sqrt{1 - \frac{1}{4Q^2}}}\frac{\omega_o}{\mathrm{GB}}$$

$$\frac{\Delta\omega_o}{\omega_o} \simeq -\frac{\omega_o}{\mathrm{GB}}, \qquad \frac{\Delta Q}{Q} \simeq 4Q\frac{\omega_o}{\mathrm{GB}}$$

Active Sensitivities

Two integrator loop architecture

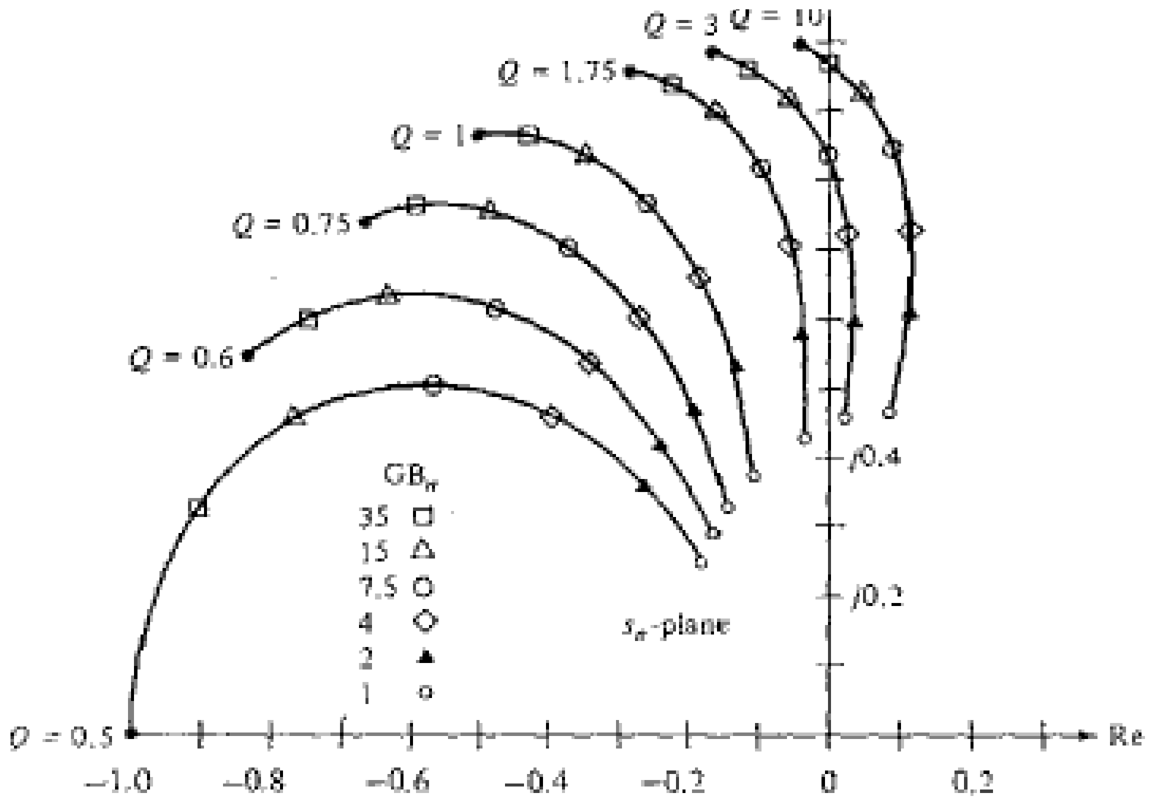


Fig. 10-17 Plot of upper half-plane root of

$$s_n^3 + s_n^2\left(\frac{1}{2}+\frac{1}{Q}+\frac{\mathrm{GB}_n}{4}\right) + s_n\frac{1}{4Q}\left(1+\mathrm{GB}_n\right) + \frac{\mathrm{GB}_n}{4} = 0$$

Active Sensitivities

- KRC

Equal-R, Equal-C

$$\omega_o = \frac{\sqrt{5 + K_0}}{RC}, \qquad Q = \frac{\sqrt{5 + K_0}}{5}$$

$$\frac{V_o}{V_i} = -\frac{\omega_o^2\left(1 - \dfrac{1}{5Q^2}\right)}{s^2 + s\dfrac{\omega_o}{Q} + \omega_o^2 + \dfrac{s}{GB}\left[s^2(25Q^2 - 4) + s\omega_o\left(20Q - \dfrac{3}{Q}\right) + \left(2 - \dfrac{1}{5Q^2}\right)\omega_o^2\right]}$$

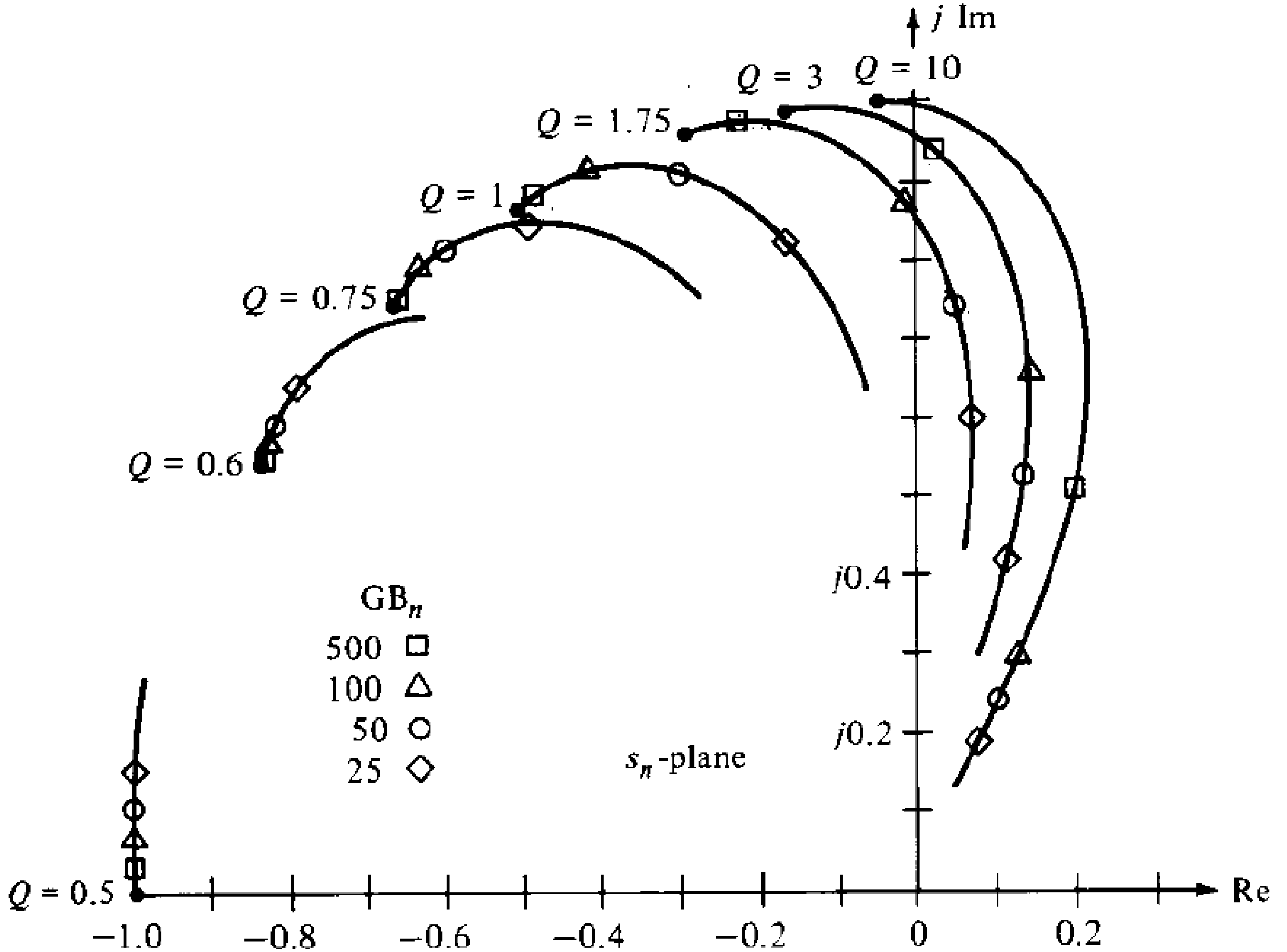
$$\left(\omega_a \ll \frac{\omega_o}{2Q}\right)$$

$$-\frac{\Delta\alpha}{\omega_o} \cong \frac{25Q^2}{2}\left(1 - \frac{1}{5Q^2}\right)\left(1 - \frac{6}{25Q^2}\right)\frac{\omega_o}{GB}, \qquad \frac{\Delta\beta}{\omega_o} \cong \frac{35Q}{4}\frac{\left(1 - \dfrac{1}{5Q^2}\right)\left(1 - \dfrac{6}{35Q^2}\right)}{\sqrt{1 - \dfrac{1}{4Q^2}}}\frac{\omega_o}{GB}$$

$$\frac{\Delta\omega_o}{\omega_o} \cong \frac{5Q}{2}\left(1 - \frac{1}{5Q^2}\right)\frac{\omega_o}{GB}, \qquad \frac{\Delta Q}{Q} \cong 25Q^3\left(1 - \frac{1}{5Q^2}\right)\left(1 - \frac{7}{5Q^2}\right)\frac{\omega_o}{GB}$$

Active Sensitivities

- KRC



Active Sensitivity Comparisons

	$\frac{\Delta\omega_0}{\omega_0}$	$\frac{\Delta Q}{Q}$
Passive RLC	NA	NA
+KRC		
Equal R, Equal C (K=3-1/Q)	$-\frac{1}{2}\left(3-\frac{1}{Q}\right)^2\tau\omega_0$	$-\frac{1}{2}\left(3-\frac{1}{Q}\right)^2\tau\omega_0$
Equal R, K=1 ($C_1=4Q^2C_2$)	$-Q\tau\omega_0$	$Q\tau\omega_0$
Bridged-T Feedback	$-\frac{3}{2}Q\tau\omega_0$	$\frac{1}{2}Q\tau\omega_0$
Two-Integrator Loop	$-\tau\omega_0$	$4Q\tau\omega_0$
-KRC	$\frac{5}{2}Q\tau\omega_0$	$25Q^3\tau\omega_0$

Substantial Differences Between Architectures

Are these passive sensitivities acceptable?

	$\left\| S_x^{\omega_0} \right\|$	$\left\| S_x^{Q} \right\|$
Passive RLC	$\leq \frac{1}{2}$	**1,1/2**
+KRC		
Equal R, Equal C (K=3-1/Q)	**0,1/2**	**Q, 2Q, 3Q**
Equal R, K=1 ($C_1=4Q^2C_2$)	**0,1/2**	**0,1/2, $2Q^2$**
Bridged-T Feedback	**0,1/2**	**1/3,1/2, 1/6**
Two-Integrator Loop	**0,1/2**	**1,1/2, 0**
-KRC	**less than or equal to 1/2**	**less than or equal to 1/2**

Are these active sensitivities acceptable?

Active Sensitivity Comparisons

	$\frac{\Delta\omega_0}{\omega_0}$	$\frac{\Delta Q}{Q}$
Passive RLC		
+KRC		
Equal R, Equal C (K=3-1/Q)	$-\frac{1}{2}\left(3-\frac{1}{Q}\right)^2\tau\omega_0$	$-\frac{1}{2}\left(3-\frac{1}{Q}\right)^2\tau\omega_0$
Equal R, K=1 ($C_1=4Q^2C_2$)	$-Q\tau\omega_0$	$Q\tau\omega_0$
Bridged-T Feedback	$-\frac{3}{2}Q\tau\omega_0$	$\frac{1}{2}Q\tau\omega_0$
Two-Integrator Loop	$-\tau\omega_0$	$4Q\tau\omega_0$
-KRC	$\frac{5}{2}Q\tau\omega_0$	$25Q^3\tau\omega_0$

Are these sensitivities acceptable?

Passive Sensitivities:

$$\frac{\Delta\omega_0}{\omega_0} \cong S_x^{\omega_0} \frac{\Delta x}{x}$$

In integrated circuits, Δ R/R and Δ C/C due to process variations can be K 30% or larger due to process variations

Many applications require $\Delta\omega_0/\omega_0$<.001 or smaller and similar requirements on $\Delta Q/Q$

Even if sensitivity is around ½ or 1, variability is often orders of magnitude too large

Active Sensitivities:

All are proportional to $\tau\omega_0$

Some architectures much more sensitive than others

Can reduce $\tau\omega_0$ by making GB large but this is at the expense of increased power and even if power is not of concern, process presents fundamental limits on how large GB can be made

What can be done to address these problems?

1. Predistortion

Design circuit so that <u>after</u> component shift, correct pole locations are obtained

Predistortion is generally used in integrated circuits to remove the bias associated with inadequate amplifier bandwidth

Predistortion does not help with process variations of passive components

Tedious process after fabrication since depends on individual components

Temperature dependence may not track

Difficult to maintain over time and temperature

Over-ordering will adversely affect performance

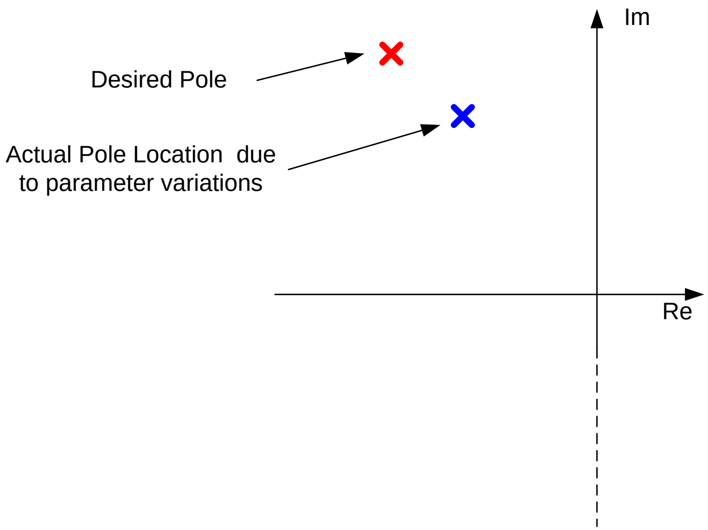
Seldom will predistortion alone be adequate to obtain acceptable performance

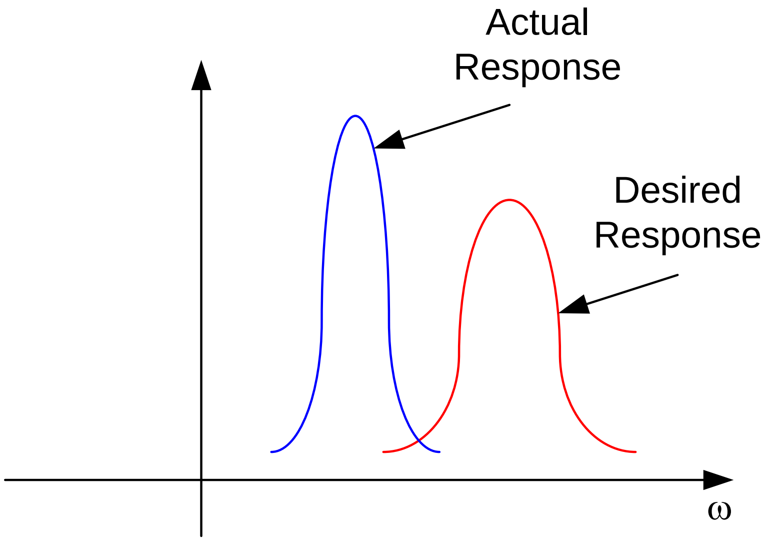
Bell Labs did to this in high-volume production (STAR Biquad)

What can be done to address these problems?

1. Predistortion

Design circuit so that after component shift, correct pole locations are obtained



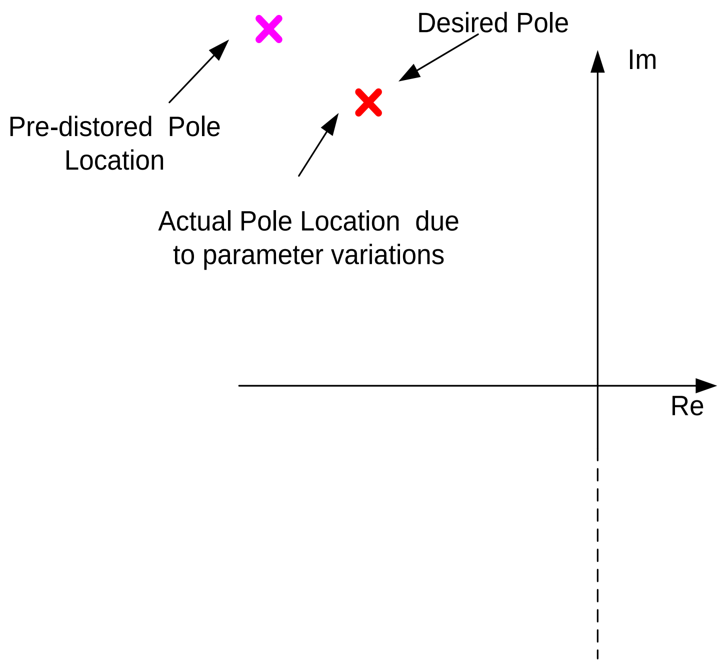


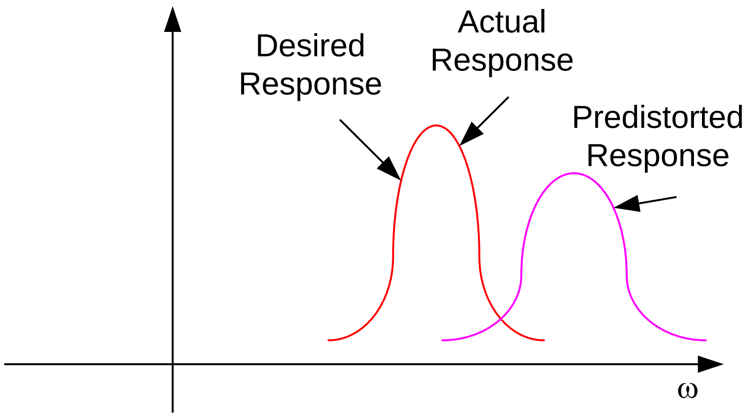
Pole shift due to parametric variations (e.g. inadequate GB)

What can be done to address these problems?

1. Predistortion

Design circuit so that <u>after</u> component shift, correct pole locations are obtained



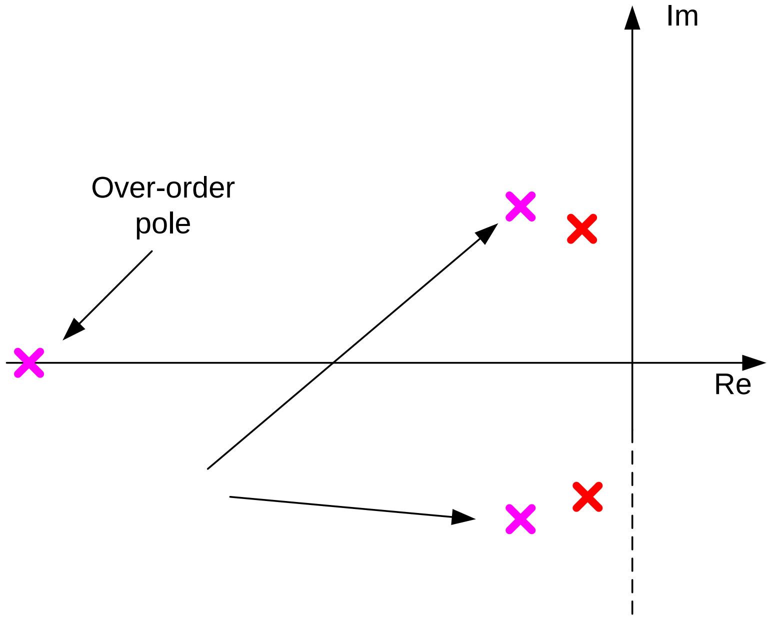


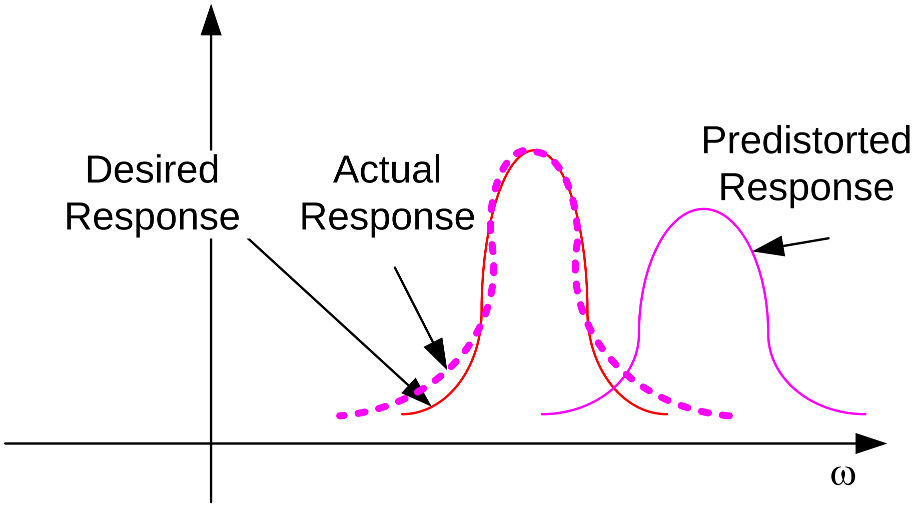
Pre-distortion concept

What can be done to address these problems?

1. Predistortion

Design circuit so that after component shift, correct pole locations are obtained





Over-ordering Limitations with Pre-distortion

Parasitic Pole Affects Response

Predistortion almost always done even if benefits only modest

Not effective if significant deviations exist before predistortion

What can be done to address these problems?

2. Trimming

a) Functional Trimming

- trim parameters of actual filter based upon measurements
- difficult to implement in many structures
- manageable for cascaded biquads

b) Deterministic Trimming (much preferred)

- Trim component values to their ideal value

  Continuous-trims of resistors possible in some special processes

  Continuous-trim of capacitors is more challenging

  Link trimming of Rs or Cs is possible with either metal or switches
- If all components are ideal, the filter should also be ideal

  R-trimming algorithms easy to implement

  Limited to unidirectional trim

  Trim generally done at wafer level for laser trimming, package for link trims
- Filter shifts occur due to stress in packaging and heat cycling

c) Master-slave reference control (depends upon matching in a process)

- Can be implemented in discrete or integrated structures
- Master typically frequency or period referenced
- Most effective in integrated form since good matching possible
- Widely used in integrated form

Stay Safe and Stay Healthy !

End of Lecture 21